UNITED STATES NAVAL ACADEMY

DIVISION OF
ENGINEERING AND WEAPONS

EXPERIMENTAL MODAL ANALYSIS OF A SANDWICH CONSTRUCTION GLASS REINFORCED PLASTIC COMPOSITE DECK PANEL

Colin P. Ratcliffe

TECHNICAL REPORT EW-10-96
JULY 1996

MECHANICAL ENGINEERING
DEPARTMENT

Dr. Colin P. Ratcliffe
Mechanical Engineering Department
United States Naval Academy
590 Holloway Road
Annapolis, MD 21402-5002
(410) 293-6535
email: ratcliff@nadn.navy.mil

EXPERIMENTAL MODAL ANALYSIS OF A SANDWICH CONSTRUCTION GLASS REINFORCED PLASTIC COMPOSITE DECK PANEL

by

Dr. Colin P. Ratcliffe

ABSTRACT

This report details the experimental modal analysis of a 20-ft by 6-ft sandwich construction glass reinforced plastic panel. The panel includes four compliant foundations, which are designed to be equipment supports. The analysis reveals that below the fundamental resonance of the foundations, the structure behaves in a similar fashion to a uniform plate. The foundations do not show appreciable vibration. Above the fundamental of the straps, there is significant sympathetic vibration between the deck and foundations.

CONTENTS

LIST OF TABLES

LIST OF FIGURES

NOMENCLATURE

ζ	Viscous Damping Ratio
f_n	Natural Frequency (Hz)
η	Loss Factor
ω	Frequency (rad-s^{-1})
Ω	Frequency Matrix
Φ	Modal Matrix
${}_r\phi_{ij}$	Modal Constant for r^{th} natural frequency (mode) between coordinates *i* and *j*.
M	Mass Matrix
K	Stiffness Matrix
m_p	Modal Mass
m	Mass
k	Stiffness
c	Damping Coefficient
x, X	Displacement, Amplitude of Displacement
F	Amplitude of Force Excitation
i	$\sqrt{-1}$
E	Young's Modulus of Elasticity
a_i	i^{th} Modal (Principle) Coordinate
${}_rA_{ij}$	Modal Constant between spatial coordinates i and j for the r^{th} natural frequency.

DEFINITION OF TERMS

dB	The decibel unit used in this report is referenced to the unit quantity measured in the SI. For example, *accelerance* is referenced to unit [1/kg] or [$m\text{-}s^{-2}N^{-1}$].
Log Mag	Where the vertical axis of graphs is labeled *Log Mag*, this indicates the units are dB.
Accelerance	Accelerance is the acceleration response per unit excitation force. It is a function of frequency, and includes both magnitude and phase information.
Receptance	Receptance is the displacement response per unit excitation force. It is a function of frequency, and includes both magnitude and phase information.
FRF	"Frequency Response Function" - a term that includes both accelerance and receptance.
Modal density	The number of natural frequencies (or resonances) in a given frequency band.
Damp (%)	This is a value shown in the natural frequency and damping results tables and is the percentage viscous damping ratio, ζ.
Modal Constant	Modal constants are a measure of the relative motion between two spatial coordinates. There is a different modal constant for every combination of coordinates, and every natural frequency. Modal constants are discussed in detail in Appendix B.

INTRODUCTION AND BACKGROUND

This report concerns the experimental modal analysis of a composite deck panel. The panel is a half-scale model of a new design. It is a sandwich construction, approximately 20 feet by 6 feet, and approximately 3-1/2 inches total thickness, with a balsa wood core and glass reinforced plastic (GRP) face sheets. Full details of the design are obtainable from the program sponsors.

The primary aim of the experimental work conducted in The Mechanical Engineering Department, United States Naval Academy, was to produce a modal analysis of the panel that could be compared to theoretical analyses conducted elsewhere.

ADMINISTRATIVE ISSUES

This report is submitted to fulfill the experimental aspects of The Order for Work and Services document N00167-96-WR-60131, dated November 1995. FRN 39169, Task Area R01234, Element 0602121N. The program sponsors are Milton Critchfield and Ronald Purcell, Naval Surface Warfare Center, Code 652, Carderock, Bethesda, MD 20084.

SUSPENSION METHOD AND BOUNDARY CONDITIONS

USNA has a well established, modern modal analysis test facility, and can test structures up to about 15-20 feet long, and 4-6 feet across in the laboratory. The equipment is portable, and has successfully been used on large structures outside the laboratory, including a 250-ft ship in dry dock.

With a panel of this size and weight, it is experimentally and fiscally infeasible to test it in the U.S.N.A. laboratory with a fixed boundary condition. Therefore it was decided to test the panel "freely suspended", and compare the results to a theoretical analysis (finite element or numerically with a Rayleigh-Ritz method) using the same boundary conditions. The theoretical analyses could be adjusted to conform to boundary conditions more appropriate for in-service conditions. In this way, there should be a minimum variation between results due to a variation of theoretical and experimental boundary conditions. The results of the theoretical analyses, and their comparison with the experimental results reported here, will be the subject of a later report. This report deals exclusively with the experimentation, and modal analysis of the experimental data.

The "free suspension" method consisted of rubber bungee cords passed under the deck panel, and secured to a steel test frame. This was to simulate as closely as possible the free boundary condition, and is a standard technique in experimental modal analysis. The suspension method introduced very low *rigid body* modes, all below about 1-2 Hz. It is believed that these modes should not alter the modal analysis to any significant extent.

Figures 1-4 show photographs of the supported deck panel. Full details of the test configuration are in Appendix A.

EXPERIMENTAL PROCEDURE

This section discusses the experimental configuration, data accuracy, and analysis accuracy for the modal testing of the deck panel. Impact force excitation with a fixed reference accelerometer was used for all these tests. The locations of the test coordinates are given in Table 1. In this table, the term *DECK* refers to the main deck panel. *MOUNT1* refers to the foundation that was tested. The other three foundations were not tested, and did not have any coordinates marked on them.

The main concern for this configuration was the choice of accelerometer location. This was located at coordinate #56, close to one edge and well away from the center-line and lines of symmetry. This position is unlikely to be on a node line at resonance, a requirement for a good location. Without repeating the experiments with the accelerometer at different test locations, it is not possible to state whether all the required natural frequencies were identified, and therefore there may be additional natural frequencies not identified in this report. The effect of this on the findings and conclusions in this report is not considered likely to be important, especially since most of the "uniform plate" modes for the deck panel were identified.

Throughout, data quality was monitored with the coherence function. Data were averaged for at least 3 impacts, and up to 5 impacts in some areas where the coherence function indicated poorer correlation between excitation and response. When testing at a particular coordinate did not give data of sufficient quality, the data were deleted, and the test at that coordinate was repeated. Overall the data quality was very good (not excellent), with the coherence generally being better than 95%. There were, however, some measurements that did not meet this high standard. The effect of this on the results is considered minimal, since the poorer signals represent a very small percentage of the total amount of measured data.

The analysis was comprehensive, and is considered to be of a high standard. Many of the modes were "split", i.e. there were 2 or more natural frequencies with almost identical mode shapes, very close in frequency. This indicates the test and analysis procedures were sufficiently accurate to identify small variations in modal behavior due to manufacturing and geometry variations. This is particularly the case for the four compliant foundations, which have similar modal behavior.

MODAL ANALYSIS PROCEDURE

The calibrated FRF data were subject to a modal analysis using a commercial STAR PC program. Appendix B contains a summary of modal analysis theory, and Appendix C summarizes the STAR program.

As indicated in Appendix B, the imaginary component of the FRF (both accelerance and receptance), will normally be a maximum at resonance. Generally, this is a better global indicator of resonance than using the magnitude, real component, or phase, of the FRF. Therefore, the data were spatially averaged $(\text{imaginary})^2$ for three separate sets of data: The deck panel by itself; the foundation by itself, and the deck and foundation combined. The three averages were made to ensure any local resonances of each sub component were identified. The spatially averaged accelerance functions $(\text{imaginary})^2$ are shown in Figures 5 and 6. The spatially averaged accelerance functions $(\text{modulus})^2$ are shown in Figures 7 and 8 for comparison.

Based on a detailed inspection of these averaged functions, several frequency bands were selected for analysis. These are detailed in Table 2. For each band, the number of natural frequencies was determined by repeating several curve fits, using different curve fit algorithms, across the complete set of 171 measured [1]FRFs. The number of natural frequencies was generally taken as the smallest number that consistently gave a good

---

[1]Only the averaged FRFs were saved to disk. There was one FRF for each of the 171 test coordinates.

reconstruction of the FRF for all 171 functions. However, it is at this stage that the "science of modal analysis" plays second fiddle to the "art of modal analysis." Experience often dictates the exact choice of frequency limits, number of natural frequencies, and optimum curve fit type.

RESULTS & DISCUSSION

Ultimately, the analysis was conducted up to 200 Hz. Thirty-one analysis bands were selected, yielding 39 natural frequencies. The frequencies and viscous damping ratios are detailed in Table 3. In this table the column marked ***Damp (%)*** gives the percentage modal viscous damping ratios, ζ_R. The modal equivalent loss factors can be found by multiplying these numbers by $\times 2$. All these figures are within the range normally expected from a conventional glass reinforced plastic structure. The modal damping is significantly higher than observed for conventional steel structures. However, it is not as high as that observed when special damping treatments or coatings are applied.

The Modal Assurance Criterion Table (MAC) is shown in Table 4. This table indicates a high standard of analysis, and is discussed further in Appendix B.

Table 5 gives an indication of the relative ranking of the various natural frequencies. For each mode, the table shows the root mean square modal constant divided by the natural frequency (in rad/s). These figures are shown separately as the average for the deck, for the foundation, and for the complete structure (deck and foundation combined). The figures in the table give some indication of the relative importance of each mode - generally the larger the number, the more significant is the natural frequency to the overall dynamic behavior of the structure. It should be noted that these figures are for guidance only. If the resonances are ranked using the numbers from this table, resonances within about four or five in the ranking may be equally important. The actual motion that would result from harmonic excitation depends on many more factors, including modal damping and modal density, and the location of excitation.

For each natural frequency up to 100 Hz (and for two above that), Figures 9-50 show a still taken from an animation of the mode shapes. There are two figures for each natural frequency - one for the foundation, and one for the main deck panel. The orientation of the components is maintained, but their relative size and location is not. This display method is chosen as the best way to show the dynamic performance of each component on a still page. Adjacent to each figure set is a brief description of the mode shape. The comments are based on a visual interpretation of the animation, and not on the stills shown in this report, which do not show the information as clearly as the animations.

The lowest natural frequencies are the plate modes of the main deck panel. At these frequencies, there is little or no independent motion of the foundations.

The first resonance of the foundation is at about 22 Hz. The motion of the foundation is such that there is identifiable sympathetic vibration of the deck (see, for example, Mode 5, Figures 17 and 18). The reason for this strong coupling is the closeness of the deck natural frequency (at 20.4 Hz) with the foundation resonances (at approximately 21.7 and 22.1 Hz).

Above the first resonance of the foundations, there is generally coupling between the foundations and deck panel. The only exceptions to this are Mode 6 (25.8 Hz, Figures 19 and 20), where the deck resonance does not significantly move the foundation, and Mode 24 (122.3 Hz, Figures 47 and 48), which is the first foundation resonance that does not unduly excite the deck.

SCALING

As described earlier, this experimental study was conducted on a half-scale model. The issue of predicting the modal behavior at full-scale is not trivial, especially when working with composite materials. It is the author's suggestion that the optimum strategy is to use these results to verify a finite element (or other theoretical) model *run at half-scale.* Once there is confidence in the model, particularly with regard to choice of elements and material properties, it is suggested that a full-scale analysis can best be performed by increasing the finite element model to full size.

CONCLUSIONS

Based on a scale of 0 (completely inadequate) to 10 (outstanding), with the range 4-6 representing normally acceptable for projects of this type, I rate the data capture at 5 (five) and the modal analysis at 8-9 (eight-nine). Combined, my estimate of overall project quality is 7 (seven), representing an above average result, more than adequate to meet the project aims.

The fundamental resonance of the deck in its "freely supported" configuration was identified at 7.55 Hz. Below approximately 22 Hz (being the first resonance of the foundations), motion of the deck and foundations was essentially independent. Above the fundamental of the foundations, there was generally strong coupling between the deck and foundations.

RECOMMENDATIONS FOR FURTHER WORK

The following recommendations for further experimental work at U.S.N.A. are made:

1. Extend the modal analysis to include all four foundations. In order to eliminate environmental effects, the best method will be to repeat the data capture for the entire deck at the same time as testing the foundations. This may include testing with varying mass loads on the foundations.

2. Investigate the transmissibility between the foundations and deck surface. This may include testing at several locations, and with varying mass loads on the foundations.

3. Repeat the modal analysis with the deck supported at its perimeter. The physical constraints of the U.S.N.A. laboratory suggest this may best be achieved by simulating a pin boundary condition by laying the deck panel on narrow edge supports. This may include testing with varying mass loads on the foundations.

....

Dr. Colin P. Ratcliffe

[2]July 1996

---

[2]The digital data are available from the author for at least two years from the date of this report.

Table 1: Coordinates.

All coordinates are in meters.

Coord #	X	Y	Z		
1	0.00	0.00	0.00	DECK	R
2	309.00e-3	0.00	0.00	DECK	R
3	618.00e-3	0.00	0.00	DECK	R
4	927.00e-3	0.00	0.00	DECK	R
5	1.24	0.00	0.00	DECK	R
6	1.54	0.00	0.00	DECK	R
7	1.85	0.00	0.00	DECK	R
8	0.00	306.00e-3	0.00	DECK	R
9	309.00e-3	306.00e-3	0.00	DECK	R
10	618.00e-3	306.00e-3	0.00	DECK	R
11	927.00e-3	306.00e-3	0.00	DECK	R
12	1.24	306.00e-3	0.00	DECK	R
13	1.54	306.00e-3	0.00	DECK	R
14	1.85	306.00e-3	0.00	DECK	R
15	0.00	612.00e-3	0.00	DECK	R
16	309.00e-3	612.00e-3	0.00	DECK	R
17	618.00e-3	612.00e-3	0.00	DECK	R
18	927.00e-3	612.00e-3	0.00	DECK	R
19	1.24	612.00e-3	0.00	DECK	R
20	1.54	612.00e-3	0.00	DECK	R
21	1.85	612.00e-3	0.00	DECK	R
22	0.00	918.00e-3	0.00	DECK	R
23	309.00e-3	918.00e-3	0.00	DECK	R
24	618.00e-3	918.00e-3	0.00	DECK	R
25	927.00e-3	918.00e-3	0.00	DECK	R
26	1.24	918.00e-3	0.00	DECK	R
27	1.54	918.00e-3	0.00	DECK	R
28	1.85	918.00e-3	0.00	DECK	R
29	0.00	1.22	0.00	DECK	R
30	309.00e-3	1.22	0.00	DECK	R
31	618.00e-3	1.22	0.00	DECK	R
32	927.00e-3	1.22	0.00	DECK	R
33	1.24	1.22	0.00	DECK	R
34	1.54	1.22	0.00	DECK	R
35	1.85	1.22	0.00	DECK	R
36	0.00	1.53	0.00	DECK	R
37	309.00e-3	1.53	0.00	DECK	R
38	618.00e-3	1.53	0.00	DECK	R
39	927.00e-3	1.53	0.00	DECK	R
40	1.24	1.53	0.00	DECK	R
41	1.54	1.53	0.00	DECK	R
42	1.85	1.53	0.00	DECK	R
43	0.00	1.84	0.00	DECK	R
44	309.00e-3	1.84	0.00	DECK	R
45	618.00e-3	1.84	0.00	DECK	R
46	927.00e-3	1.84	0.00	DECK	R
47	1.24	1.84	0.00	DECK	R
48	1.54	1.84	0.00	DECK	R
49	1.85	1.84	0.00	DECK	R
50	0.00	2.14	0.00	DECK	R
51	309.00e-3	2.14	0.00	DECK	R
52	618.00e-3	2.14	0.00	DECK	R
53	927.00e-3	2.14	0.00	DECK	R
54	1.24	2.14	0.00	DECK	R
55	1.54	2.14	0.00	DECK	R

56	1.85	2.14	0.00	DECK	R
57	0.00	2.45	0.00	DECK	R
58	309.00e-3	2.45	0.00	DECK	R
59	618.00e-3	2.45	0.00	DECK	R
60	927.00e-3	2.45	0.00	DECK	R
61	1.24	2.45	0.00	DECK	R
62	1.54	2.45	0.00	DECK	R
63	1.85	2.45	0.00	DECK	R
64	0.00	2.75	0.00	DECK	R
65	309.00e-3	2.75	0.00	DECK	R
66	618.00e-3	2.75	0.00	DECK	R
67	927.00e-3	2.75	0.00	DECK	R
68	1.24	2.75	0.00	DECK	R
69	1.54	2.75	0.00	DECK	R
70	1.85	2.75	0.00	DECK	R
71	0.00	3.06	0.00	DECK	R
72	309.00e-3	3.06	0.00	DECK	R
73	618.00e-3	3.06	0.00	DECK	R
74	927.00e-3	3.06	0.00	DECK	R
75	1.24	3.06	0.00	DECK	R
76	1.54	3.06	0.00	DECK	R
77	1.85	3.06	0.00	DECK	R
78	0.00	3.37	0.00	DECK	R
79	309.00e-3	3.37	0.00	DECK	R
80	618.00e-3	3.37	0.00	DECK	R
81	927.00e-3	3.37	0.00	DECK	R
82	1.24	3.37	0.00	DECK	R
83	1.54	3.37	0.00	DECK	R
84	1.85	3.37	0.00	DECK	R
85	0.00	3.67	0.00	DECK	R
86	309.00e-3	3.67	0.00	DECK	R
87	618.00e-3	3.67	0.00	DECK	R
88	927.00e-3	3.67	0.00	DECK	R
89	1.24	3.67	0.00	DECK	R
90	1.54	3.67	0.00	DECK	R
91	1.85	3.67	0.00	DECK	R
92	0.00	3.98	0.00	DECK	R
93	309.00e-3	3.98	0.00	DECK	R
94	618.00e-3	3.98	0.00	DECK	R
95	927.00e-3	3.98	0.00	DECK	R
96	1.24	3.98	0.00	DECK	R
97	1.54	3.98	0.00	DECK	R
98	1.85	3.98	0.00	DECK	R
99	0.00	4.28	0.00	DECK	R
100	309.00e-3	4.28	0.00	DECK	R
101	618.00e-3	4.28	0.00	DECK	R
102	927.00e-3	4.28	0.00	DECK	R
103	1.24	4.28	0.00	DECK	R
104	1.54	4.28	0.00	DECK	R
105	1.85	4.28	0.00	DECK	R
106	0.00	4.59	0.00	DECK	R
107	309.00e-3	4.59	0.00	DECK	R
108	618.00e-3	4.59	0.00	DECK	R
109	927.00e-3	4.59	0.00	DECK	R
110	1.24	4.59	0.00	DECK	R
111	1.54	4.59	0.00	DECK	R
112	1.85	4.59	0.00	DECK	R
113	0.00	4.90	0.00	DECK	R
114	309.00e-3	4.90	0.00	DECK	R
115	618.00e-3	4.90	0.00	DECK	R
116	927.00e-3	4.90	0.00	DECK	R
117	1.24	4.90	0.00	DECK	R
118	1.54	4.90	0.00	DECK	R
119	1.85	4.90	0.00	DECK	R
120	0.00	5.20	0.00	DECK	R

121	309.00e-3	5.20	0.00	DECK	R
122	618.00e-3	5.20	0.00	DECK	R
123	927.00e-3	5.20	0.00	DECK	R
124	1.24	5.20	0.00	DECK	R
125	1.54	5.20	0.00	DECK	R
126	1.85	5.20	0.00	DECK	R
127	0.00	5.51	0.00	DECK	R
128	309.00e-3	5.51	0.00	DECK	R
129	618.00e-3	5.51	0.00	DECK	R
130	927.00e-3	5.51	0.00	DECK	R
131	1.24	5.51	0.00	DECK	R
132	1.54	5.51	0.00	DECK	R
133	1.85	5.51	0.00	DECK	R
134	0.00	5.81	0.00	DECK	R
135	309.00e-3	5.81	0.00	DECK	R
136	618.00e-3	5.81	0.00	DECK	R
137	927.00e-3	5.81	0.00	DECK	R
138	1.24	5.81	0.00	DECK	R
139	1.54	5.81	0.00	DECK	R
140	1.85	5.81	0.00	DECK	R
141	0.00	6.12	0.00	DECK	R
142	309.00e-3	6.12	0.00	DECK	R
143	618.00e-3	6.12	0.00	DECK	R
144	927.00e-3	6.12	0.00	DECK	R
145	1.24	6.12	0.00	DECK	R
146	1.54	6.12	0.00	DECK	R
147	1.85	6.12	0.00	DECK	R
148	480.00e-3	710.00e-3	150.00e-3	MOUNT1	R
149	608.57e-3	710.00e-3	150.00e-3	MOUNT1	R
150	737.14e-3	710.00e-3	150.00e-3	MOUNT1	R
151	865.71e-3	710.00e-3	150.00e-3	MOUNT1	R
152	994.29e-3	710.00e-3	150.00e-3	MOUNT1	R
153	1.12	710.00e-3	150.00e-3	MOUNT1	R
154	1.25	710.00e-3	150.00e-3	MOUNT1	R
155	1.38	710.00e-3	150.00e-3	MOUNT1	R
156	480.00e-3	804.00e-3	150.00e-3	MOUNT1	R
157	608.57e-3	804.00e-3	150.00e-3	MOUNT1	R
158	737.14e-3	804.00e-3	150.00e-3	MOUNT1	R
159	865.71e-3	804.00e-3	150.00e-3	MOUNT1	R
160	994.29e-3	804.00e-3	150.00e-3	MOUNT1	R
161	1.12	804.00e-3	150.00e-3	MOUNT1	R
162	1.25	804.00e-3	150.00e-3	MOUNT1	R
163	1.38	804.00e-3	150.00e-3	MOUNT1	R
164	480.00e-3	898.00e-3	150.00e-3	MOUNT1	R
165	608.57e-3	898.00e-3	150.00e-3	MOUNT1	R
166	737.14e-3	898.00e-3	150.00e-3	MOUNT1	R
167	865.71e-3	898.00e-3	150.00e-3	MOUNT1	R
168	994.29e-3	898.00e-3	150.00e-3	MOUNT1	R
169	1.12	898.00e-3	150.00e-3	MOUNT1	R
170	1.25	898.00e-3	150.00e-3	MOUNT1	R
171	1.38	898.00e-3	150.00e-3	MOUNT1	R

Note: *"MOUNT1"* refers to the compliant foundation.

Table 2: Modal Curve Fit Information.

This table shows the settings used for curve fitting. The cursor bands are defined by their Frequency range (from lower to upper), and show the data used for each curve fit. The mode # identifies the natural frequency numbers, and the number of natural frequencies curve fit in each cursor band. Curve fit types are described in Appendix C. *Polynomial* refers to the single pass curve fit. *Global F&D* or *Global Res* identifies the two pass process was used. The number of extra terms used was the STAR default. For more information, refer to the STAR manual.

Frequencies are in Hz.

Cursor Band	Lower Frequency	Upper Frequency	Low Mode #	High Mode #	Curve Fit Type	Extra Terms
1	6.88	8.44	1.00	1.00	Global Res	4
2	11.25	12.81	2.00	2.00	Global Res	4
3	19.06	23.13	3.00	5.00	Global Res	4
4	25.00	26.56	6.00	6.00	Global Res	4
5	38.75	40.94	7.00	8.00	Global Res	4
6	41.88	44.06	9.00	10.00	Global Res	4
7	48.44	50.63	11.00	11.00	Global Res	4
8	62.50	64.06	12.00	12.00	Global Res	4
9	65.31	66.88	13.00	13.00	Global Res	4
10	70.94	72.50	14.00	14.00	Global Res	4
11	84.38	87.19	15.00	17.00	Global Res	4
12	89.38	90.94	18.00	18.00	Global Res	4
13	94.06	95.63	19.00	19.00	Global Res	4
14	101.25	102.81	20.00	20.00	Global Res	4
15	103.75	105.31	21.00	21.00	Global Res	4
16	108.44	110.00	22.00	22.00	Global Res	4
17	116.56	118.75	23.00	23.00	Global Res	4
18	121.88	123.44	24.00	24.00	Global Res	4
19	124.38	125.94	25.00	25.00	Global Res	4
20	126.88	128.44	26.00	26.00	Global Res	4
21	131.88	133.44	27.00	27.00	Global Res	4
22	141.56	145.31	28.00	28.00	Global Res	4
23	148.75	150.31	29.00	29.00	Global Res	4
24	150.94	153.13	30.00	30.00	Global Res	4
25	161.88	164.38	31.00	32.00	Global Res	4
26	165.63	167.19	33.00	33.00	Global Res	4
27	179.38	182.81	34.00	35.00	Global Res	4
28	185.94	187.50	36.00	36.00	Global Res	4
29	193.44	196.25	37.00	37.00	Global Res	4
30	198.75	200.31	38.00	38.00	Global Res	4
31	200.63	202.19	39.00	39.00	Global Res	4

Table 3: Natural Frequencies and Viscous Damping Ratios.

This table shows the estimated natural frequencies and viscous damping ratios up to about 200 Hz. The column ***Mode #*** identifies the natural frequency number. ***Damp (%)*** is the percentage viscous damping ratio.

Mode #	**Freq**	**Damp (%)**
1	7.55	3.01
2	11.87	2.14
3	20.42	1.42
4	21.67	1.07
5	22.10	1.11
6	25.76	1.59
7	39.24	631.12e-3
8	40.72	676.56e-3
9	42.33	870.67e-3
10	42.69	1.20
11	49.62	895.57e-3
12	63.16	956.09e-3
13	65.76	735.11e-3
14	71.33	701.88e-3
15	84.98	479.63e-3
16	85.79	481.76e-3
17	86.32	387.36e-3
18	90.18	572.46e-3
19	94.67	587.23e-3
20	101.84	739.54e-3
21	104.56	908.58e-3
22	109.03	698.14e-3
23	117.46	611.12e-3
24	122.26	547.47e-3
25	124.32	550.62e-3
26	127.29	420.03e-3
27	132.41	724.69e-3
28	143.86	1.64
29	149.52	584.23e-3
30	151.34	527.39e-3
31	164.00	1.15
32	164.25	469.58e-3
33	166.34	621.23e-3
34	179.83	481.60e-3
35	181.76	661.74e-3
36	186.65	801.83e-3
37	194.34	722.82e-3
38	200.46	668.63e-3
39	201.33	443.32e-3

Table 4: Modal Assurance Criterion (MAC) Table.

Columns 1 - 20

1.00	0.00	0.00	0.07	0.05	0.00	0.00	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01
0.00	1.00	0.00	0.00	0.00	0.00	0.01	0.01	0.04	0.02	0.00	0.00	0.00	0.00	0.00	0.02	0.02	0.01	0.00	0.00
0.00	0.00	1.00	0.01	0.42	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.07	0.00	0.01	1.00	0.62	0.00	0.00	0.00	0.00	0.00	0.06	0.00	0.00	0.04	0.00	0.00	0.00	0.00	0.00	0.00
0.05	0.00	0.42	0.62	1.00	0.00	0.00	0.00	0.00	0.00	0.04	0.00	0.01	0.03	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	1.00	0.01	0.00	0.02	0.02	0.00	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.01	0.01	0.00	0.00	0.01	1.00	0.16	0.09	0.04	0.00	0.00	0.00	0.00	0.02	0.04	0.04	0.03	0.01	0.00
0.02	0.01	0.00	0.00	0.00	0.00	0.16	1.00	0.08	0.04	0.00	0.00	0.00	0.00	0.03	0.04	0.03	0.03	0.02	0.00
0.00	0.04	0.00	0.00	0.00	0.02	0.09	0.08	1.00	0.73	0.00	0.00	0.00	0.00	0.54	0.38	0.29	0.15	0.00	0.00
0.00	0.02	0.00	0.00	0.00	0.02	0.04	0.04	0.73	1.00	0.00	0.00	0.00	0.00	0.47	0.28	0.21	0.10	0.00	0.00
0.00	0.00	0.00	0.06	0.04	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.01	0.31	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.02	0.00	0.00	0.00	0.00	0.00	1.00	0.01	0.00	0.00	0.01	0.01	0.00	0.00	0.00
0.00	0.00	0.02	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.01	0.01	1.00	0.03	0.00	0.00	0.00	0.00	0.01	0.00
0.00	0.00	0.00	0.04	0.03	0.00	0.00	0.00	0.00	0.00	0.31	0.00	0.03	1.00	0.01	0.01	0.01	0.00	0.01	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.03	0.54	0.47	0.00	0.00	0.00	0.01	1.00	0.51	0.38	0.20	0.00	0.00
0.00	0.02	0.00	0.00	0.00	0.00	0.04	0.04	0.38	0.28	0.00	0.01	0.00	0.01	0.51	1.00	0.96	0.84	0.00	0.00
0.00	0.02	0.00	0.00	0.00	0.00	0.04	0.03	0.29	0.21	0.00	0.01	0.00	0.01	0.38	0.96	1.00	0.92	0.00	0.00
0.00	0.01	0.00	0.00	0.00	0.00	0.03	0.03	0.15	0.10	0.00	0.00	0.00	0.00	0.20	0.84	0.92	1.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.02	0.00	0.00	0.00	0.00	0.01	0.01	0.00	0.00	0.00	0.00	1.00	0.00
0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.04
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.03	0.00	0.01	0.00	0.00	0.01	0.01	0.01	0.01	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.02	0.00	0.00	0.01	0.02	0.02	0.01	0.00	0.00
0.01	0.00	0.00	0.07	0.05	0.00	0.00	0.00	0.01	0.00	0.03	0.00	0.03	0.02	0.01	0.00	0.01	0.00	0.08	0.00
0.01	0.00	0.01	0.02	0.01	0.00	0.00	0.00	0.00	0.00	0.02	0.00	0.02	0.02	0.00	0.00	0.00	0.00	0.08	0.00
0.01	0.00	0.00	0.13	0.08	0.00	0.00	0.00	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.00
0.00	0.00	0.04	0.00	0.02	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.02	0.00	0.00	0.00	0.00	0.04	0.00
0.01	0.00	0.00	0.00	0.00	0.00	0.02	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01
0.00	0.02	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.02	0.00	0.00	0.00	0.00	0.00	0.04	0.05	0.07	0.00	0.00
0.00	0.00	0.00	0.03	0.03	0.00	0.00	0.00	0.00	0.00	0.08	0.00	0.00	0.03	0.00	0.00	0.01	0.01	0.00	0.00
0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.03	0.00	0.00	0.00	0.00	0.00	0.03	0.03	0.01	0.01	0.02	0.00	0.00	0.05	0.01	0.00	0.00	0.00	0.01	0.01
0.03	0.00	0.00	0.00	0.00	0.00	0.02	0.03	0.00	0.00	0.02	0.00	0.00	0.06	0.00	0.00	0.00	0.00	0.01	0.01
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.06	0.00	0.00	0.30	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.03	0.00	0.00	0.01	0.01	0.00	0.06	0.00	0.01	0.00	0.00	0.00	0.01	0.00	0.00
0.01	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.03
0.00	0.00	0.00	0.02	0.01	0.00	0.00	0.00	0.02	0.01	0.15	0.00	0.00	0.34	0.02	0.02	0.01	0.01	0.00	0.00
0.00	0.00	0.01	0.03	0.02	0.00	0.00	0.00	0.01	0.01	0.11	0.00	0.01	0.34	0.01	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.01	0.05	0.02	0.00	0.01	0.00	0.01	0.00	0.20	0.00	0.00	0.15	0.01	0.00	0.01	0.00	0.00	0.00

Columns 21-39

0.00	0.00	0.00	0.01	0.01	0.01	0.00	0.01	0.00	0.00	0.00	0.03	0.03	0.00	0.00	0.01	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.01	0.00	0.04	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01
0.00	0.00	0.00	0.07	0.02	0.13	0.00	0.00	0.00	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.03	0.05
0.00	0.00	0.00	0.05	0.01	0.08	0.02	0.00	0.00	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.02	0.02
0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.01	0.00	0.00	0.03	0.02	0.00	0.00	0.01	0.00	0.00	0.01
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.00	0.00	0.00	0.03	0.03	0.00	0.00	0.01	0.00	0.00	0.00
0.00	0.03	0.00	0.01	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.01	0.00	0.02	0.01	0.01
0.00	0.03	0.00	0.00	0.00	0.01	0.00	0.00	0.02	0.00	0.00	0.01	0.00	0.00	0.01	0.00	0.01	0.01	0.00
0.00	0.00	0.00	0.03	0.02	0.00	0.01	0.00	0.00	0.08	0.00	0.02	0.02	0.06	0.00	0.00	0.15	0.11	0.20
0.00	0.01	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.06	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.03	0.02	0.00	0.00	0.00	0.00	0.00	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00
0.00	0.00	0.00	0.02	0.02	0.00	0.02	0.00	0.00	0.03	0.00	0.05	0.06	0.30	0.01	0.00	0.34	0.34	0.15
0.00	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.02	0.01	0.01
0.00	0.01	0.02	0.00	0.00	0.00	0.00	0.00	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.00	0.00
0.00	0.01	0.02	0.01	0.00	0.00	0.00	0.00	0.05	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.01
0.00	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.07	0.01	0.00	0.00	0.00	0.00	0.01	0.00	0.01	0.00	0.00
0.02	0.00	0.00	0.08	0.08	0.03	0.04	0.00	0.00	0.00	0.00	0.01	0.01	0.00	0.00	0.02	0.00	0.00	0.00
0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.01	0.01	0.00	0.00	0.03	0.00	0.00	0.00
1.00	0.00	0.00	0.05	0.03	0.01	0.07	0.00	0.00	0.00	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01
0.00	1.00	0.00	0.03	0.01	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.04	0.00	0.00	0.00	0.01	0.00
0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.05	0.00	0.00	0.00	0.00
0.05	0.03	0.00	1.00	0.77	0.33	0.32	0.00	0.00	0.02	0.01	0.01	0.01	0.04	0.00	0.00	0.01	0.00	0.02
0.03	0.01	0.00	0.77	1.00	0.22	0.20	0.00	0.00	0.02	0.00	0.00	0.01	0.05	0.00	0.00	0.00	0.00	0.02
0.01	0.00	0.00	0.33	0.22	1.00	0.16	0.00	0.00	0.02	0.00	0.00	0.00	0.04	0.00	0.00	0.01	0.02	0.02
0.07	0.00	0.00	0.32	0.20	0.16	1.00	0.00	0.00	0.02	0.03	0.02	0.01	0.02	0.00	0.00	0.00	0.02	0.03
0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.02	0.01	0.00	0.01	0.11	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.11	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00
0.00	0.01	0.00	0.02	0.02	0.02	0.02	0.00	0.11	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.06	0.14	0.11
0.04	0.00	0.00	0.01	0.00	0.00	0.03	0.00	0.00	0.00	1.00	0.34	0.23	0.01	0.00	0.01	0.00	0.01	0.01
0.00	0.00	0.00	0.01	0.00	0.00	0.02	0.02	0.00	0.00	0.34	1.00	0.83	0.07	0.00	0.05	0.08	0.02	0.01
0.00	0.00	0.00	0.01	0.01	0.00	0.01	0.01	0.00	0.00	0.23	0.83	1.00	0.07	0.00	0.03	0.06	0.02	0.01
0.00	0.04	0.00	0.04	0.05	0.04	0.02	0.00	0.00	0.00	0.01	0.07	0.07	1.00	0.02	0.01	0.16	0.15	0.14
0.00	0.00	0.05	0.00	0.00	0.00	0.00	0.01	0.01	0.00	0.00	0.00	0.00	0.02	1.00	0.01	0.00	0.01	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.11	0.00	0.00	0.01	0.05	0.03	0.01	0.01	1.00	0.01	0.00	0.00
0.00	0.00	0.00	0.01	0.00	0.01	0.00	0.00	0.00	0.06	0.00	0.08	0.06	0.16	0.00	0.01	1.00	0.38	0.22
0.00	0.01	0.00	0.00	0.00	0.02	0.02	0.00	0.00	0.14	0.01	0.02	0.02	0.15	0.01	0.00	0.38	1.00	0.68
0.01	0.00	0.00	0.02	0.02	0.02	0.03	0.00	0.00	0.11	0.01	0.01	0.01	0.14	0.00	0.00	0.22	0.68	1.00

Table 5: Average Modal Constants.

This table gives the root mean square modal constant normalized to the circular natural frequency. Averages are shown separately for the deck, for the foundation, and for the complete structure. The figures give some indication of the relative importance of each mode - generally the larger the number, the more significant is the natural frequency. It should be noted that these figures are for guidance only. If the resonances are ranked using these numbers, resonances within about four or five in the ranking may be equally important. The actual motion that would result from harmonic excitation depends on many more factors, including modal damping, modal density, and the location of excitation.

Mode #	Frequency (Hz)	Avg Constant (Deck)	Avg Constant (Foundation)	Avg Constant (Combined)	(units are $kg^{-1}s^{-1}$)
1	7.545	0.0410	0.0013	0.0355	
2	11.866	0.0593	0.0016	0.0512	
3	20.42	0.0505	0.0015	0.0436	
4	21.669	0.0022	0.0003	0.0019	
5	22.101	0.0188	0.0009	0.0163	
6	25.76	0.0830	0.0019	0.0716	
7	39.24	0.0024	0.0003	0.0021	
8	40.724	0.0047	0.0005	0.0041	
9	42.333	0.0227	0.0010	0.0196	
10	42.687	0.1023	0.0021	0.0883	
11	49.617	0.0003	0.0001	0.0002	
12	63.156	0.0375	0.0013	0.0324	
13	65.761	0.0747	0.0018	0.0645	
14	71.328	0.0002	0.0001	0.0001	
15	84.976	0.0228	0.0010	0.0197	
16	85.791	0.0095	0.0006	0.0082	
17	86.318	0.0207	0.0010	0.0180	
18	90.183	0.0560	0.0016	0.0484	
19	94.671	0.0575	0.0016	0.0496	
20	101.837	0.0504	0.0015	0.0435	
21	104.558	0.0921	0.0020	0.0795	
22	109.03	0.0017	0.0003	0.0015	
23	117.462	0.0991	0.0021	0.0855	
24	122.26	0.0358	0.0013	0.0310	
25	124.318	0.0366	0.0013	0.0316	
26	127.293	0.0043	0.0004	0.0037	
27	132.412	0.0464	0.0014	0.0401	
28	143.861	0.0574	0.0016	0.0495	
29	149.515	0.0143	0.0008	0.0124	
30	151.335	0.0014	0.0003	0.0013	
31	163.996	0.2291	0.0032	0.1974	
32	164.246	0.0099	0.0007	0.0086	
33	166.339	0.1416	0.0025	0.1221	
34	179.834	0.0056	0.0005	0.0049	
35	181.764	0.1706	0.0027	0.1471	
36	186.652	0.1974	0.0029	0.1701	
37	194.344	0.0044	0.0004	0.0038	
38	200.455	0.0541	0.0015	0.0467	
39	201.325	0.0587	0.0016	0.0507	

Figure 1: General View of the Suspended Deck Panel.

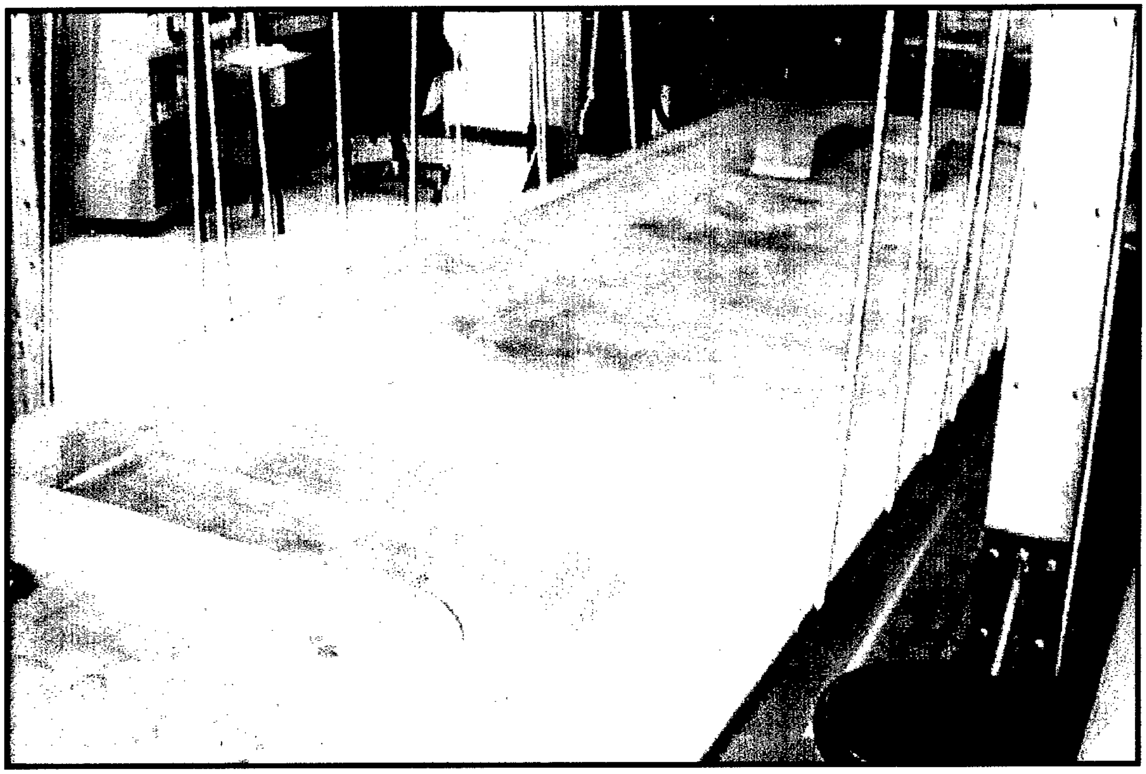


Figure 2: The Suspended Deck Panel.

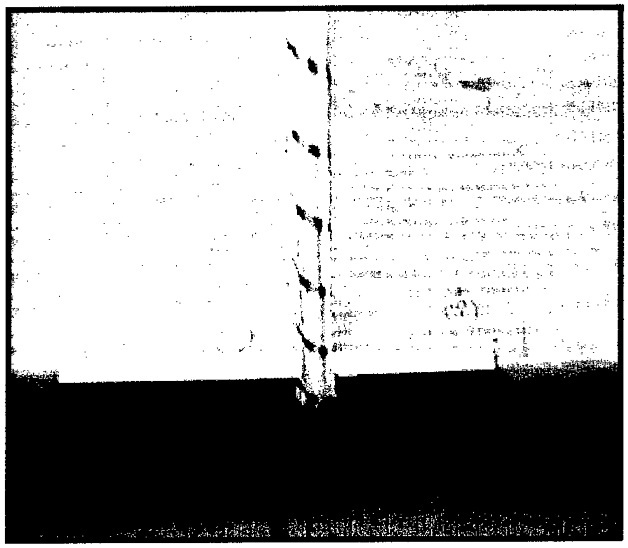


Figure 3: Rubber Bungee and
Aluminum Protection Strip.

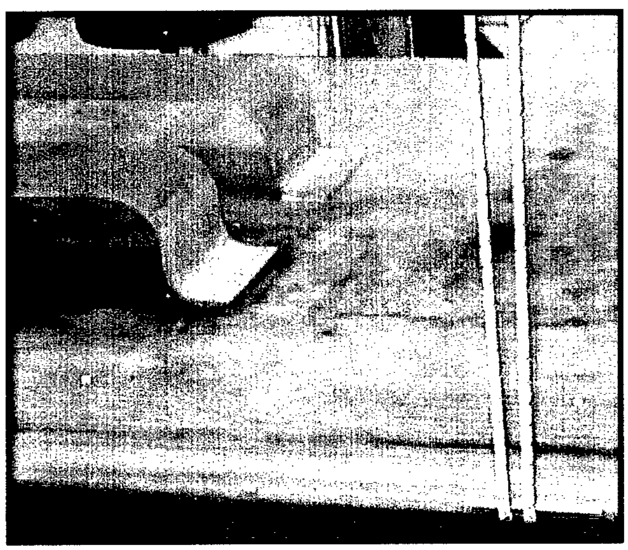


Figure 4: Rubber Bungee, and part of the
Compliant Foundations.

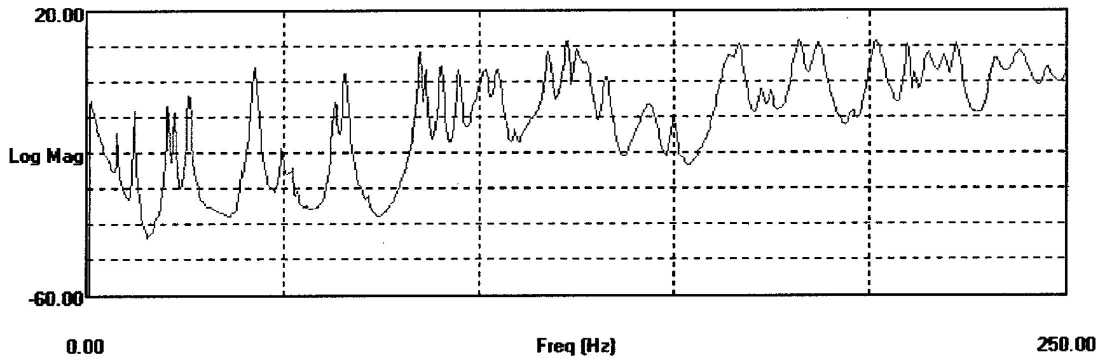


Figure 5: Spatially Averaged Accelerance (imaginary)^2 -- whole structure.

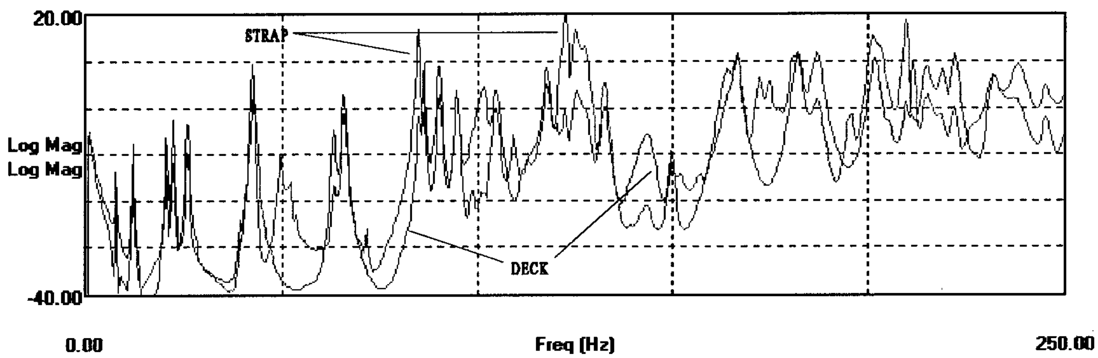


Figure 6: Spatially Averaged Accelerance (imaginary)^2 - deck & foundation comparison.

These figures show the data primarily used to identify natural frequencies. Additionally, it can be seen in the lower figure that generally the foundation has a higher (worse) accelerance.

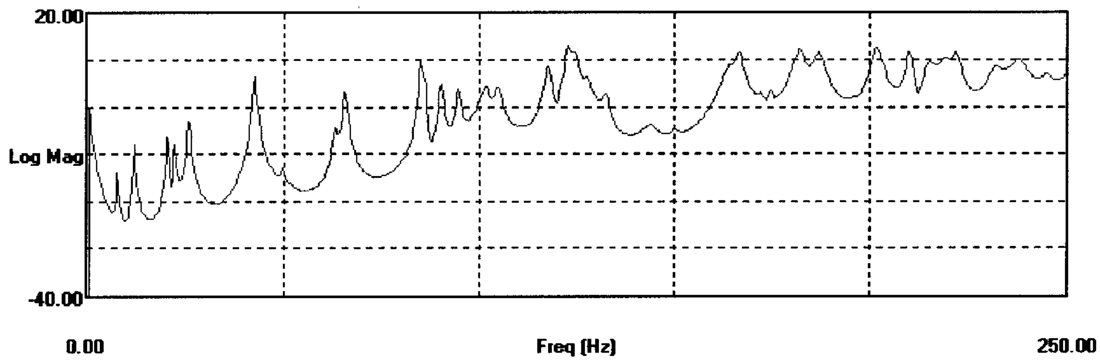


Figure 7: Spatially Averaged accelerance (modulus)^2 -- whole structure.

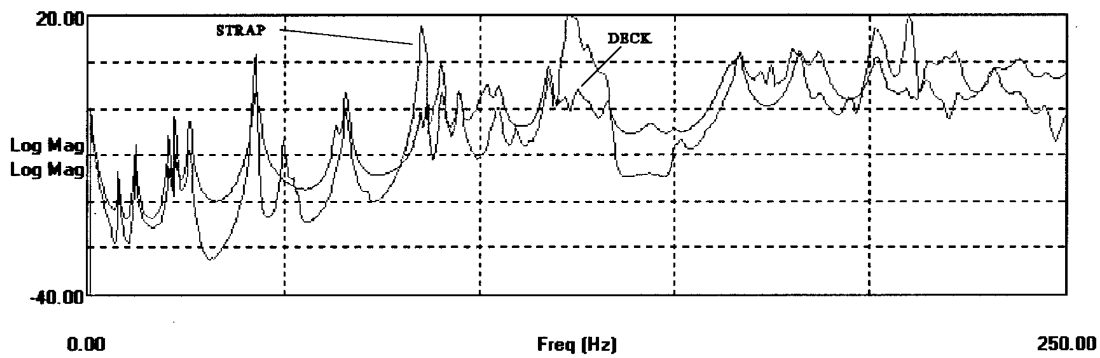
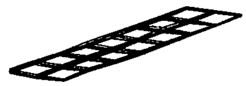


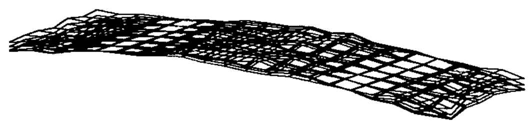
Figure 8: Spatially Averaged Accelerance (modulus)^2 - deck & foundation comparison.

These figures give an indication of the actual response of the structure, as a function of frequency. The lower figure confirms the earlier finding that the foundation tends to be more resonant than the deck.

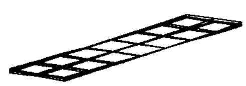
Figures 9 & 10: Mode 1: 7.55 Hz, 3%



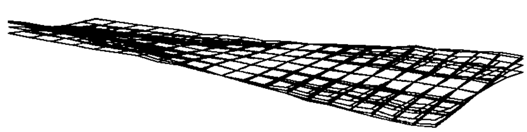
This is the strong fundamental resonance of the deck, acting much like a beam. The foundation has little or no independent vibration.



Figures 11 & 12: Mode 2: 11.9 Hz, 2%

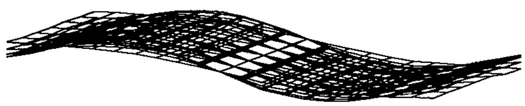


This is the first "torsional" vibration of the deck. Again, the foundation has little or no independent vibration.

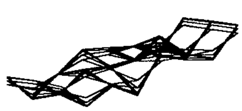


Figures 13 & 14: Mode 3: 20.4 Hz, 1.4%

This is the strong, second "beam" resonance of the deck. The foundation is showing slight motion.



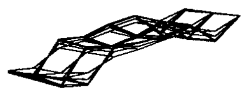
Figures 15 & 16: Mode 4: 21.7 Hz, 1.1%

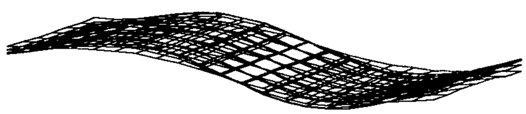


This is primarily motion of the foundation. There is some coupling to the deck.

Figures 17 & 18: Mode 5: 22.1 Hz, 1.1%

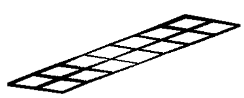
Strong motion predominantly of the foundation, with strong coupling to the deck.

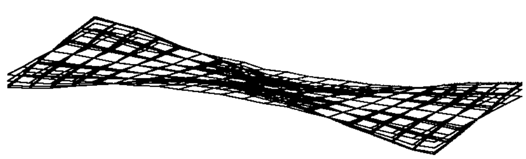




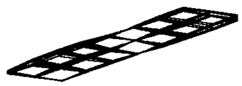
Figures 19 & 20: Mode 6: 25.8 Hz, 1.6%

A strong "torsional" resonance of the deck. The foundation has little or no independent vibration.

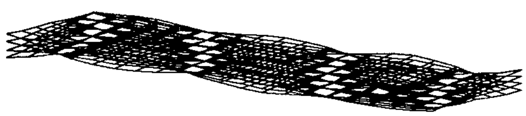




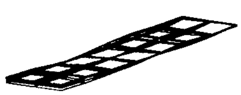
Figures 21 & 22: Mode 7: 39.2 Hz, 0.6%



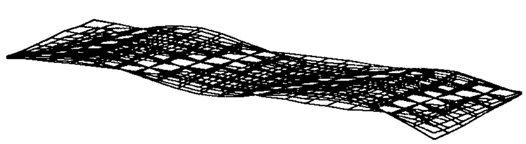
This is a relatively weak resonance, with comparable motion of both the foundation and deck.



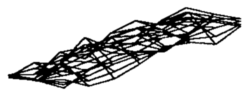
Figures 23 & 24: Mode 8: 40.7 Hz, 0.7%



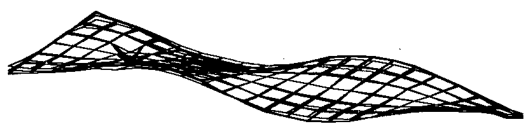
This is a relatively weak resonance, with comparable motion of both the foundation and deck.



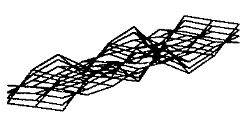
Figures 25 & 26: Mode 9: 42.3 Hz, 0.9%



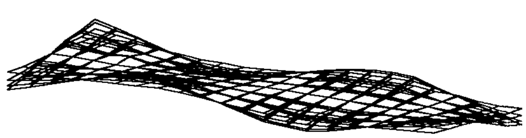
This is a strong resonance of the deck, with strong coupling to the foundation.



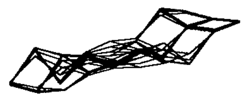
Figures 27 & 28: Mode 10: 42.7 Hz, 1.2%



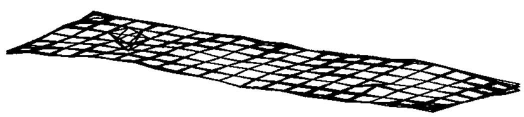
This is a strong resonance of the foundation, with strong coupling to the deck.



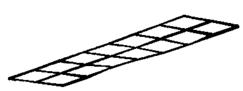
Figures 29 & 30: Mode 11: 49.6 Hz, 0.9%



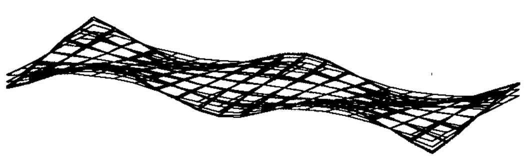
This is the lowest frequency resonance of the foundation that does not significantly couple with the deck.



Figures 31 & 32: Mode 12: 63.2 Hz, 1.0%

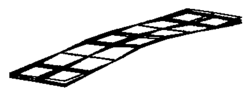


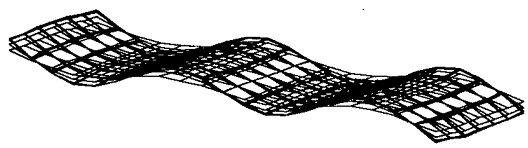
Strong resonance of the deck, with only slight motion of the foundation.



Figures 33 & 34: Mode 13: 65.8 Hz, 0.7%

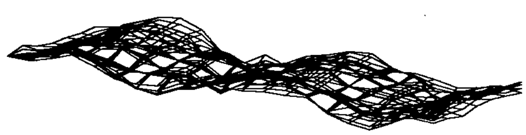
This beam-like resonance of the deck is strongly coupled with a resonance of the foundation.





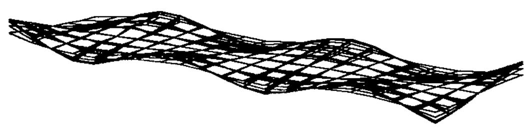
Figures 35 & 36: Mode 14: 71.3 Hz, 0.7%

This is probably a sympathetic resonance due to the resonance of another foundation. From the measured data it does not appear to be significant. However if another foundation is resonating, the resonance may be significant.



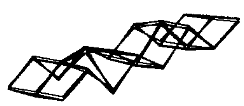
Figures 37 & 38: Mode 15: 85.0 Hz, 0.5%

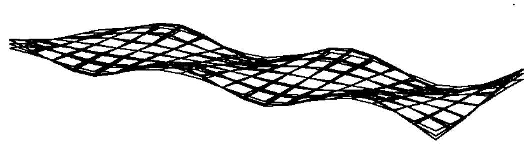
Strong resonance of the foundation, coupling with the deck.



Figures 39 & 40: Mode 16: 85.8 Hz, 0.5%

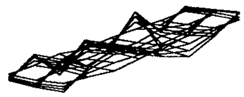
Moderate resonance of the foundation, with strong coupling with the deck.

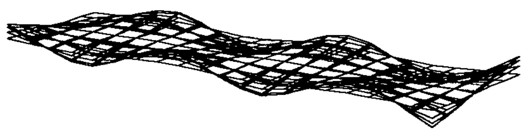




Figures 41 & 42: Mode 17: 86.3 Hz, 0.4%

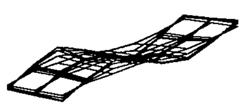
Moderate resonance of the foundation, with strong coupling with the deck.

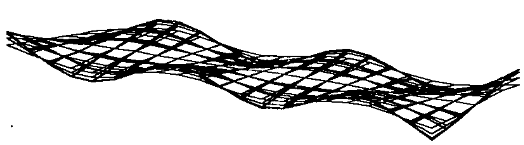




Figures 43 & 44: Mode 18: 90.2 Hz, 0.6%

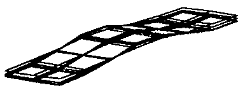
Moderate resonance of the foundation, with strong coupling with the deck.

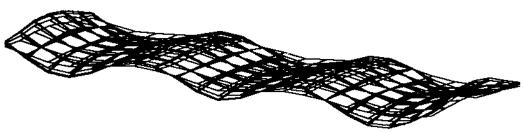




Figures 45 & 46: Mode 19: 94.7 Hz, 0.6%

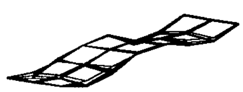
A significant resonance, with strong coupling between the deck and foundation.

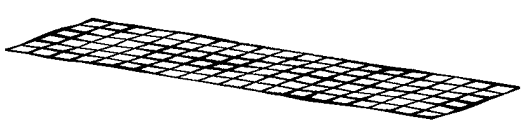




Figures 47 & 48: Mode 24: 122.3 Hz, 0.5%

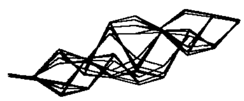
A strong resonance of the foundation that does not couple with the deck

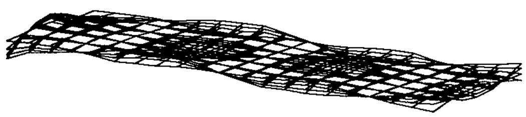




Figures 49 & 50: Mode 25: 124.3 Hz, 0.6%

A strong resonance of the foundation, with moderate coupling with the deck.





APPENDIX A

EXPERIMENTAL CONFIGURATION FOR IMPACT EXCITATION TEST

APPENDIX A

EXPERIMENTAL CONFIGURATION FOR IMPACT EXCITATION TESTS

The deck panel was suspended on rubber bungee cords, passed across the 6-feet dimension of the plate. A total of 22 vertical cords was used, giving a load in each bungee in the range 20-30 (lbs). Experience dictates that this load range in these particular bungee cords is optimum for providing a very low suspension natural frequency, well below the structural modes of the panel. Schematics of the test configuration are shown in Figures A-1 and A-2, and photographs of the suspended deck are shown in Figures A-3 and A-4. The edge contact points between the rubber and deck were protected with bent aluminum strips, as shown in the photograph in Figure A-5. The strips were necessary because the sharp edges of the deck tended to cut into the rubber bungees. The effect of the aluminum strips was not quantified, but is believed to be very small.

Impact force excitation with a fixed reference accelerometer was used for all tests. The low-frequency accelerometer Type PCB 353M168 #19608 was secured with bees wax to coordinate #56, close to one edge and well away from the center-line and lines of symmetry.

Each coordinate was impacted in turn, with a calibrated PCB 086C20 #8834 force hammer. The conditioned acceleration and force gauge signals were captured, Fourier Transformed and divided by a Hewlett Packard 3562A analyzer, and the resulting accelerance functions were downloaded to PC for analysis with the STAR modal analysis package. Throughout, data quality was monitored with the coherence function. Data were averaged for at least 3 impacts, and up to 5 impacts in some areas where the coherence function indicated poorer correlation between excitation and response. Data from poor impacts (e.g. double blows, etc.) were immediately excluded from the analysis. Overall, data quality was very good (not excellent), with the coherence generally being better than 95%. There were, however, some measurements that did not meet this high standard. The effect of this on the results is considered minimal, since the poorer signals represent a very small percentage of the total amount of measured data.

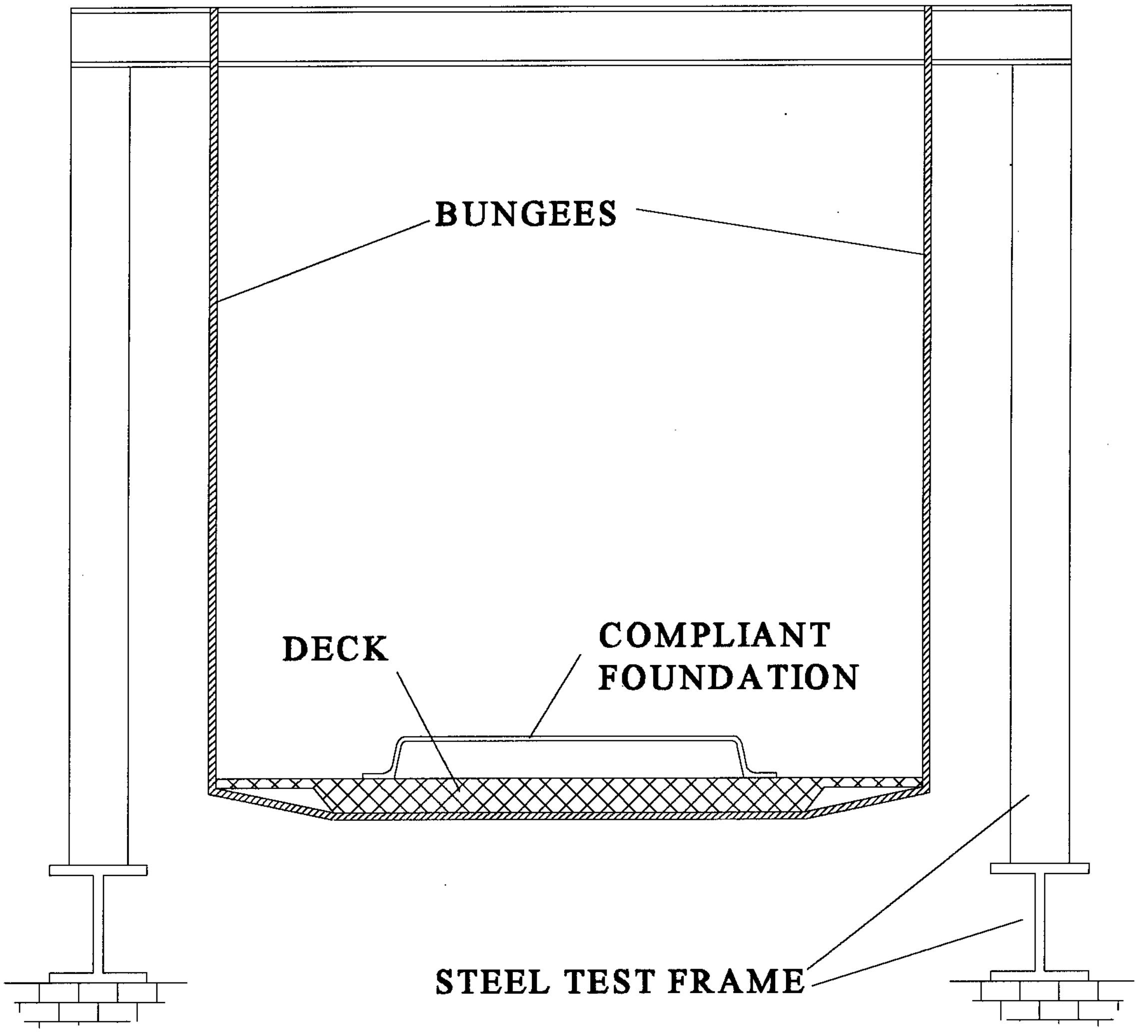


Figure A-1: Schematic of the Test Configuration (end view).

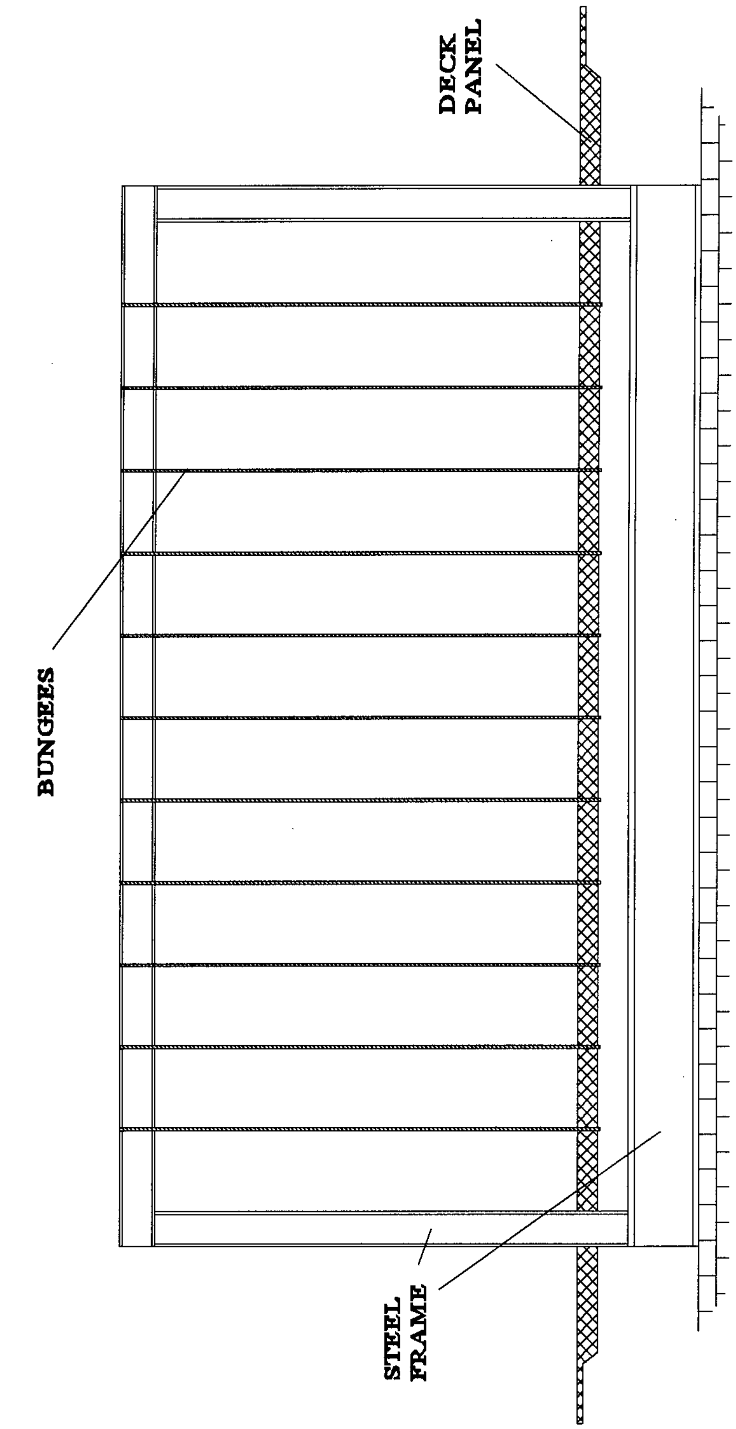


Figure A-2: Schematic of the Test Configuration (Elevation).

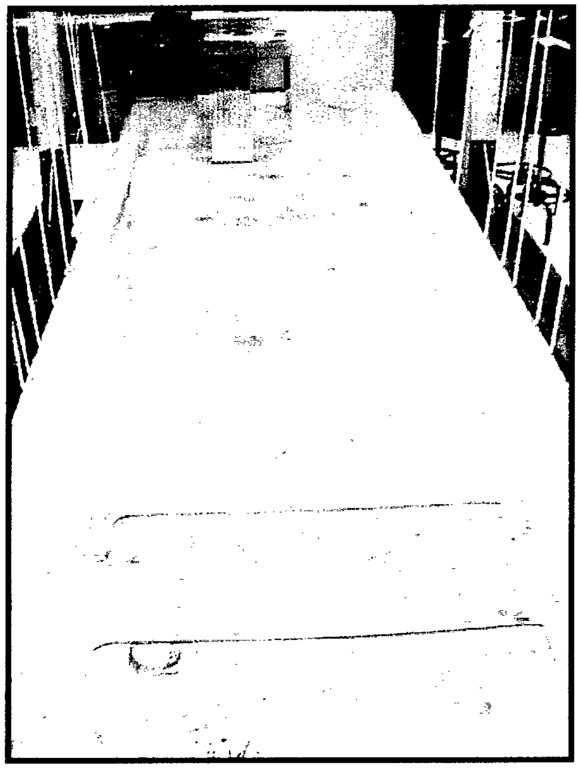


Figure A-3: General View.

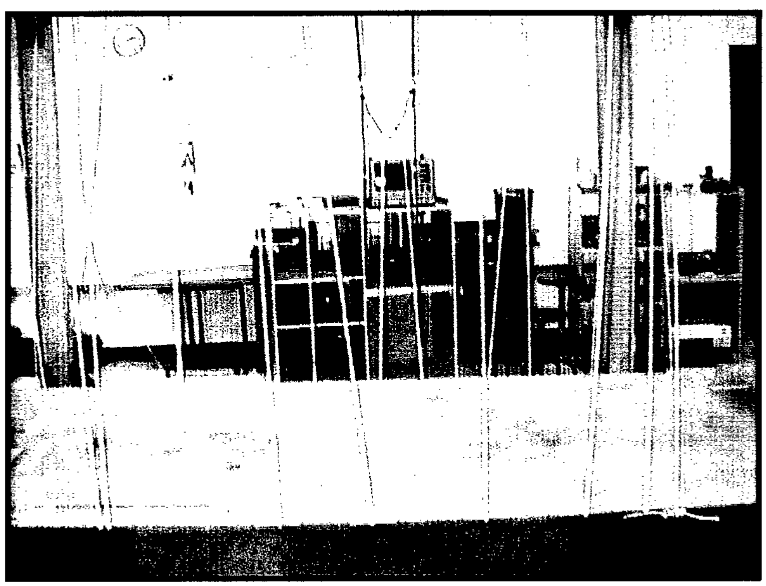


Figure A-4: General View.

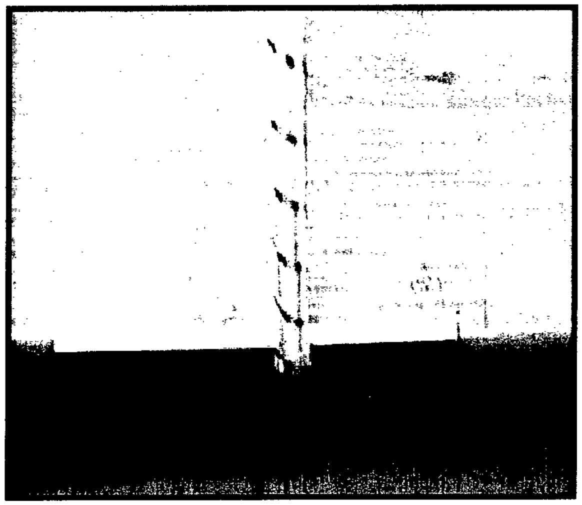


Figure A-5: Detail of Rubber Bungee and Aluminum Edge Strip.

APPENDIX B

THEORY OF MODAL ANALYSIS

APPENDIX B

THEORY OF MODAL ANALYSIS

This appendix presents an overview of the elementary theory of modal analysis. A more detailed study of the matrix algebra of mechanical vibrations is available in many standard texts, including [Inman][1]. An in depth study of experimental modal analysis is available in [Ewins][2]. The following are included in this appendix:

INTRODUCTION

Time signals can be Fourier Transformed to frequency signals (the Fourier series). Both time and frequency signals contain the same information, but present it in a different way. Similarly, structural response is in the time domain (for example, we can observe the response on an oscilloscope) and it can be transformed to the frequency domain. Selecting the appropriate transformation gives a series of *modal responses,* with one term in the series for each natural frequency of the structure. For simple components, a natural frequency can be defined as the frequency at which the structure will vibrate freely after it is given some initial excitation, providing there is little or no energy dissipation.

All structures have an infinite number of natural frequencies, but a modal analysis is normally limited to the first 20 or so. There are several reasons for this, the most important for this project being that as frequency increases, so can modal density (i.e. natural frequencies get closer together). It can be argued that when modal density is high, modal analysis is an inappropriate analysis tool. This is not considered a problem for this project, since it is predominantly the low frequency behavior which is required.

When a structure is vibrating freely, it deflects to a certain shape, with the amplitude of motion changing harmonically at the natural frequency. This deflected shape is termed the *mode shape.* Mode shapes may be complex, i.e. they include phase information. There is a different mode shape for each natural frequency.

BACKGROUND TO MODAL ANALYSIS

Modal analysis is an experimental method of finding a mathematical model that describes the dynamic behavior of a structure. Although it relies heavily on Newton's Laws of Motion, it is a state-of-the-art method. One of the earliest useable references [Kennedy & Pancu, 1947][3] theoretically developed a curve fit method still in use today, although at that time the required data accuracy could not be achieved experimentally. During the period 1960-70, development of theoretical algorithms was limited by functionality of the hardware. From 1970-1990, hardware improvements (especially the advent of good quality digital systems) meant the improved quality of measured data gave a better understanding of the theory and required algorithms. During this period, there were rapid advances in both modal analysis theory and data measurement methods.

Today, modal analysis can be considered a fully functional, general purpose, experimental tool, with new applications constantly being developed.

What does modal analysis give?

The final product of a modal analysis is an estimate of the natural frequencies of a structure. For each natural frequency, the analysis gives an estimate of damping, or energy dissipation efficiency. Where data are gathered over a sufficient number of spatial coordinates, the analysis also gives the mode shapes. The shapes are calculated as amplitude and phase of response for each location tested.

Combined, this information presents a mathematical model which has the same dynamic behavior as the structure. Since the model defines the structure, it should not depend on the test method used for the experiments.

DAMPING (ENERGY DISSIPATION) MODELS

There are two main damping (or energy dissipation) models used for modal analysis. They are viscous and hysteretic damping.

Viscous damping. (viscous damping ratio, ζ)

With viscous damping the force restricting motion is proportional to velocity, and is given by:

$$f_c(t) = c\,\dot{x}(t)$$

c is a constant, called the *damping coefficient*, which has units of [N-s/m] or [kg/s]. For fluid filled applications, c can be determined from fluid principles, but for other applications f_c is provided by equivalent effects occurring in material deformation. Theoretical viscous damping is applicable to all types of motion, including transient and steady state harmonic.

Rather than the dimensional quantity, c, which depends on specific applications, the non dimensional viscous damping ratio, ζ, is usually considered. ζ is zero when a system is undamped and $\zeta = 1$ when the system is critically damped. ζ is defined as the ratio of damping coefficient to that required for critical damping:

$$\zeta = \frac{c}{c_{critical}} = \frac{c}{2\sqrt{k\,m}}$$

where m and k are the mass and stiffness of the single degree of freedom system respectively. Examples of the unforced response of a single degree of freedom system for $\zeta = 1$ and $\zeta = 0.40$ are shown in Figure B-1. Solutions for $\zeta > 1$ are not harmonic, and are not considered in this report.

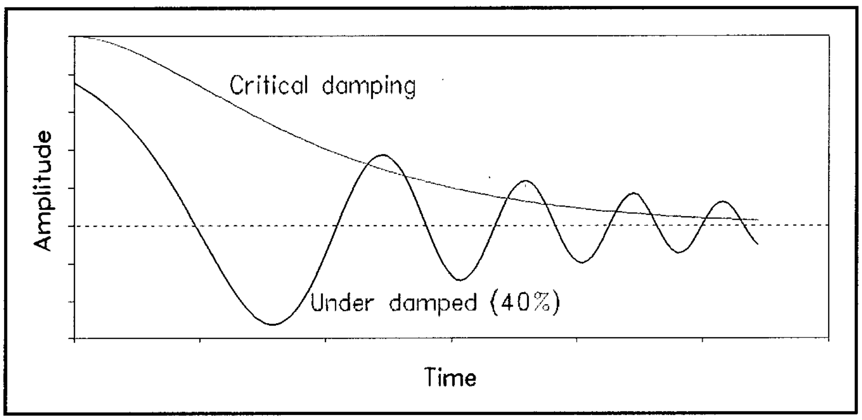


Figure B-1: Varying viscous damping ratio for a single degree of freedom system.

Hysteretic damping. (loss factor, η)

A problem with theoretical viscous damping is that for a constant amplitude of motion, the force and energy dissipation rate increase with frequency. Hysteretic damping is more representative of the damping mechanisms found in many structures, with dissipation remaining constant with frequency. The amount of hysteretic damping (loss factor) can be calculated from a hysteresis plot of force v. displacement. Loss factor is only applicable to steady state harmonic excitation, and is defined as:

$$\eta = \frac{\text{(energy lost per radian)}}{\text{(peak potential energy)}}$$

At resonance, the equivalent viscous damping ratio and loss factor (i.e. the ones that give the same energy dissipation) are related by:

$$(\textit{loss factor}) = 2 \times (\textit{viscous damping ratio})$$

$$\eta = 2 \times \zeta$$

Complex Young's Modulus of Elasticity.

When using hysteretic damping and harmonic excitation at circular frequency ω, the equation of motion for a single degree of freedom system is:

$$\left\{ k\,(1 + i\,\eta) - m\,\omega^2 \right\} X \sin(\omega t) = F \sin(\omega t)$$

This equation is identical to the equation of motion for an undamped system, but with the real stiffness term replaced with the complex stiffness term

$$k\,(1 + i\,\eta)$$

This gives rise to the notion of a complex Young's Modulus of Elasticity, which includes both the stiffness and damping effects of the material.

$$E^* = E\,(1 + i\eta)$$

RESPONSE OF A SINGLE DEGREE OF FREEDOM SYSTEM TO HARMONIC FORCE EXCITATION

A modal decomposition (as described later in this appendix) reduces a multi-degrees of freedom system to several single degree of freedom oscillators. This section therefore reviews the essential aspects of a single degree of freedom oscillator subject to harmonic force excitation.

The equation of motion (hysteretic damping) is:

$$m\ddot{x} + k(1 + i\eta)x = F\sin(\omega t)$$

For which the general solution is:

$$x(t) = X\sin(\omega t)$$

Substituting into the equation of motion and rearranging yields the equation that defines the Frequency Response Function (FRF) for a single degree of freedom oscillator:

$$\frac{X}{F} = \frac{1}{m\left\{(\omega_n^2 - \omega^2) + i\eta\omega_n^2\right\}}$$

The equivalent FRF for viscous damping is:

$$\frac{X}{F} = \frac{1}{m\left\{(\omega_n^2 - \omega^2) + 2i\zeta\omega\omega_n\right\}}$$

MULTI DEGREES OF FREEDOM

For a discrete system with many degrees of freedom, the equation of motion is presented in matrix form:

$$[M]\,\ddot{\underline{x}} + [K]\,\underline{x} = \underline{f}(t)$$

where $\underline{f}(t)$ is a vector of forcing functions, and $\underline{x}$ is a vector of responses. [M] and [K] are the mass and stiffness matrices respectively. Hysteretic damping is assumed in this equation, in which case matrix [K] is complex.

The natural frequencies and associated mode shapes for the system are found by solving for $\underline{f}(t)$ = zero and substituting

$$\underline{x} = \underline{\phi_r} \sin(\omega_r t)$$

hence

$$\left[K - \omega_r^2 M\right] \underline{\phi_r} = 0$$

or

$$\left[M^{-1} K\right] \underline{\phi_r} = \omega_r^2\, \underline{\phi_r}$$

This is the eigenvalue problem, which can be solved using standard methods. The eigenvalues, ω_r^2, represent the natural frequencies:

(eigenvalue) = ω_r^2 = (natural frequency in rad/s)^2

The eigenvectors are identical to the mode shapes.

Frequency and Modal Matrices (Eigenstructure).

It is common practice to present the eigenstructure into two matrices, called the frequency matrix and the modal matrix. The frequency matrix, Ω, is a diagonal matrix of the eigenvalues:

$$\Omega^2 = \begin{bmatrix} \omega_1^2 & & & & \\ & \omega_2^2 & & & \\ & & \cdot & & \\ & & & \cdot & \\ & & & & \omega_N^2 \end{bmatrix}$$

The modal matrix is a full matrix whose columns are the eigenvectors:

$$[\Phi] = \begin{bmatrix} {}_1\phi_1 & {}_2\phi_1 & \cdot & \cdot \\ {}_1\phi_2 & {}_2\phi_2 & \cdot & \cdot \\ {}_1\phi_3 & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ {}_1\phi_N & \cdot & \cdot & {}_N\phi_N \end{bmatrix}$$

where the term ${}_r\phi_i$ indicates the i-th element of the r-th eigenvector.

Orthogonality and Normalization.

It can be shown that the eigenvectors (mode shapes) are orthogonal with respect to both the mass and stiffness matrices:

$$[\Phi]^T[M][\Phi] = m_P[I]$$

$$[\Phi]^T[K][\Phi] = m_P[\Omega^2]$$

where m_p is the *modal mass*. This is an arbitrary number, used to scale, or normalize, the eigenvectors. Orthogonality is a fundamental requirement of modal analysis, however experimental mode shapes may not be completely orthogonal. This is discussed in the section on the Modal Assurance Criterion (MAC) table on Page B-19.

Modal Decomposition (Decoupling the Equations of Motion).

The (N×N) matrix equation of motion represents N separate equations. Each equation includes several different spatial coordinates, which means the equations are coupled, and mathematically are complicated to solve. Decoupling means applying a coordinate transformation, such that each new equation only has one independent coordinate. The new coordinates are called the **modal coordinates** (or principal coordinates). These relate directly to the natural frequencies of a structure. There is one coordinate for each resonance. In the following exposition it is assumed that the modal mass is unity. This simplifies the equations, without introducing any new restrictions.

Repeating the equation of motion:

$$[M]\,\underline{\ddot{x}} + [K]\,\underline{x} = \underline{f}(t)$$

The coordinate transformation used relates the spatial coordinates, $\underline{x}$, to the modal coordinates, $\underline{a}$, by using the modal matrix.

$$\underline{x} = [\Phi]\,\underline{a}$$

Applying this substitution, and premultiplying the equation by the transpose of the modal matrix yields:

$$[\Phi]^T[M][\Phi]\underline{\ddot{a}} + [\Phi]^T[K][\Phi]\underline{a} = [\Phi]^T \underline{f}(t)$$

Orthogonality, as described on Page B-10, is used to replace the left side of this equation with diagonal matrices:

$$\left[\Omega^2 - \omega^2[I]\right]\underline{a} = \underline{f_g}(t)$$

where $\underline{f}_g(t) = [\Phi]^T\underline{f}(t)$ is called the generalized force vector. The above equation now represents the N independent equations:

$$\begin{aligned} (\omega_1^2 - \omega^2)a_1 &= f_{g_1} \\ (\omega_2^2 - \omega^2)a_2 &= f_{g_2} \\ &\cdots\cdots\cdots \\ (\omega_N^2 - \omega^2)a_N &= f_{g_N} \end{aligned}$$

This represents the modal decomposition of the N spatial coordinates $x_1, x_2, \ldots x_N$ into the N modal (principal) coordinates $a_1, a_2, \ldots a_N$.

Single Point Excitation and Response.

A full modal analysis requires numerous transfer functions measured between the spatial coordinates on a structure. To do this, normal experimentation and data collection methods employ single point excitation and single point response. This procedure is either repeated across the structure, or data are gathered simultaneously if hardware permits. When the previous analysis is restricted to single point excitation at coordinate # j, the spatial force vector becomes:

$$f(t) = \begin{bmatrix} 0 \\ .. \\ .. \\ f_j \\ .. \\ 0 \end{bmatrix}$$

If this vector is incorporated into the matrix equation, and further restricting our interest to the response at coordinate #i, the series giving the transfer function between the i-th and j-th spatial coordinates (for hysteretic damping) reduces to:

$$\frac{X_i}{F_j} = \sum_{r=1}^{N} \frac{{}_r\phi_i \cdot {}_r\phi_j}{m_r \left\{ \omega_r^2 - \omega^2 + i\eta_r \omega_r^2 \right\}}$$

This equation is the summation of N single degree of freedom systems (compare the equations on Page B-8), and represents the combination of the

N modal coordinate responses. This is the inverse of the modal decomposition discussed previously.

Modal Constant.

A *modal constant* defines the relative motion (magnitude and phase) between two spatial coordinates on a structure. There is a different modal constant for every combination of excitation and response coordinates, and for every natural frequency. This means a modal analysis will generate a large number of modal constants. For example, in the analysis of 20 natural frequencies for a structure with 171 spatial coordinates, there will be 20 × 171 = 3420 modal constants generated. This assumes reciprocity, which significantly reduces the number of different modal constants from the theoretical maximum number of $20 \times 171^2 = 584{,}820$.

From the series:

$$\frac{X_i}{F_j} = \sum_{r=1}^{N} \frac{{}_r\phi_i \cdot {}_r\phi_j}{m_r\left\{\omega_r^2 - \omega^2 + i\,\eta_r\,\omega_r^2\right\}}$$

the modal constant is defined as:

$${}_rA_{ij} = \frac{{}_r\phi_i \cdot {}_r\phi_j}{m_r}$$

${}_rA_{ij}$ is the modal constant between spatial coordinates i and j for the r-th natural frequency. Reciprocity assumes that ${}_rA_{ij} = {}_rA_{ji}$. Substituting for ${}_rA_{ij}$ reduces the above series to:

$$\frac{X_i}{F_j} = \sum_{r=1}^{N} \frac{{}_rA_{ij}}{\left\{\omega_r^2 - \omega^2 + i\eta_r\omega_r^2\right\}}$$

This is the basic equation used for curve fitting the experimental data.

CURVE FITTING

There are numerous curve fit algorithms, each with its own advantages and disadvantages. Most curve fit algorithms are effective with well behaved and good quality data, however the potential of dynamic coupling between the deck panel and the attached foundations increases the complexity of the analysis. Therefore, great care had to be taken in this project in order to ensure a suitable choice and application of the curve fits was made. The two curve fit algorithms used for this project (using their *STAR* names) were **polynomial** and Global Frequency & Damping **(GF&D)**.

The polynomial method uses the complex data set close to each resonance, and employs a least squares approximation to determine the natural frequency, modal viscous damping ratio and modal constant. The method can estimate several (or just one) natural frequencies in the frequency band of analysis, and is therefore referred to as a MDOF method. This is a *single pass* method, where the natural frequency, damping and modal constant are estimated for each transfer function in turn. Transfer functions for different spatial coordinates are analyzed sequentially.

The GF&D method is a *two pass* method. Since the natural frequency and damping are properties of the structure, they should not depend on the test coordinates. This requirement is used by the GF&D curve fit method during the first pass. The transfer functions for all the pairs of measurement coordinates are analyzed to determine the optimum natural frequency and damping for the frequency analysis band. In this way, all the transfer functions measured spatially across the structure contribute to these estimates. In the second pass, each transfer function is analyzed in turn, and the modal constant is determined using the previously calculated natural frequency and damping values.

The GF&D method can be very powerful where there are two or more natural frequencies nearly the same, and when the natural frequencies are known not to change during a test. The natural frequencies may change if, for example, a transducer is moved across the structure during the test (change of mass loading), or if the stiffness of the structure is temperature sensitive and the test conditions vary. The tests for this project were of relatively short duration, and there was no variation in mass loading.

Symmetrical structures (for example, cylinders, circular disks and square plates) theoretically can have several identical natural frequencies. When this is transcribed to a real structure, the slight variations in size and shape caused by, for example, manufacturing variations, can make the structure have two or more natural frequencies that are very close together, rather than identical. This may be the situation for the compliant foundations on this structure, since the four foundations are nominally the same. Therefore, it is anticipated that there may be some very closely spaced natural frequencies. The effect on this project is discussed more in the section on the Modal Assurance Criterion (MAC) Table on Page B-19.

Because of this potential difficulty with the data, the analysis procedure for each frequency band was therefore to use a polynomial curve fit (a relatively fast fit method) to assist in determining the number of natural frequencies in each band. Once this was decided, the analysis was repeated using the GF&D method in order to obtain the best possible modal parameter estimates.

ACCURACY EXPECTATIONS OF MODAL PARAMETERS

The accuracy of modal data is highly dependent on the vibration characteristics of the structure, the accuracy of the data, and signal noise during measurement. The following is a summary of the accuracies expected for the results reported for this project. Note that this is not a full scientific error analysis, but is subjective and based on the observations of a highly experienced researcher (me!).

In accordance with my personal standard operating procedures, all the data for the deck were reanalyzed specifically for this report. This intense analysis period helps improve the accuracy of the final product.

Natural Frequencies. The quoted natural frequencies are assumed accurate to within approximately ±1% (±1 Hz in 100 Hz). Where there are two very close natural frequencies, the *relative accuracy* is better, and is estimated at approximately ±0.5%.

Viscous Damping Ratio. The estimate is that percentage viscous damping ratios are accurate to one decimal place at best. This is an *overall* estimate of accuracy, and individual values may be significantly more or less accurate.

Modal Constants. The estimate of accuracy is ±3%. This is an *overall* estimate of accuracy, and individual values may be significantly more or less accurate.

Animation of the mode shapes helps to identify individual modal coordinates that are of poorer accuracy, since they do not follow a "smooth" transition between nearby coordinates. In this case, the accuracy is reduced from the figure estimated here.

MODAL ASSURANCE CRITERION TABLE (MAC)

The MAC table is an estimate of the orthogonality between mode shapes. It is a square matrix, formed using the principles of orthogonality described on Page B-10. If all the estimated modes are orthogonal, the MAC table degenerates to the identity matrix. Off-diagonal elements in the table vary from zero (mode shapes are completely orthogonal) to one (mode shapes are identical). Note that the leading diagonal elements are always one.

Lack of Orthogonality

While the MAC table is a guide to the overall quality of data and analysis, the MAC for structures such as the deck for this project can apparently erroneously identify problems. There are some instances in the reported results where entries on the tri-diagonal of the matrix are substantially non zero. Consider, for example, the 16th, 17th, and 18th natural frequencies. An extract from the MAC table is shown below. This shows the inter-modal MAC values at 96%, 84% and 92%.

	Mode 16	Mode 17	Mode 18
Mode 16	1.00	0.96	0.84
Mode 17	0.96	1.00	0.92
Mode 18	0.84	0.92	1.00

We need to consider what is causing this 84-96% similarity between the mode shapes. The deck has four similar foundations built onto it, although data were only measured for one of them. With this geometric similarity, natural frequencies of the foundations may be very close together and the mode shapes measured on the deck (as opposed to the foundations) may be very similar. This is because the foundation resonances are essentially "local" resonances (the foundations and deck panel vibrate almost independently at certain frequencies), but there will always be some connectivity causing the deck to vibrate. The similarity between the deflected shapes of the deck at the natural frequencies of the foundations causes entries in the MAC table to rise. Therefore, for the results given in this report, high entries in the MAC table may indicate predominantly foundation activity, rather than poor data or analysis.

REFERENCES

1. Inman D.J., [1994], Engineering Vibration. Prentice Hall.

2. Ewins D.J., [1985], Modal Testing: Theory and Practice. Mechanical Engineering Research Studies, Engineering Dynamics Series, Brüel and Kjær.

3. Kennedy and Pancu, [1947], "Use of vectors in vibration measurement and analysis", Journal of the Aeronautical Society.

APPENDIX C

STAR

MODAL ANALYSIS PROGRAM

APPENDIX C

STAR MODAL ANALYSIS PROGRAM

The primary analysis procedure used for this project was modal analysis. An outline of the theory of modal analysis is in Appendix B. This appendix gives brief details of the STAR modal analysis software. This is commercial software, available from Genrad. Full details of the software are available in the program user and reference manuals[1].

The STAR System is a series of Windows compatible software products for testing and analyzing the dynamics of mechanical structures. STAR is an acronym for Structural Testing, Analysis and Reporting, which describes the general capabilities of the system. The system is designed to meet both the needs of the light user with occasional testing requirements, and the heavy user with a dedicated testing laboratory. For this project, the only STAR System product used was STAR*Modal*®. This program uses the frequency response function (FRF) method to identify the modal properties of a mechanical structure. STAR*Modal*® acquires measurements from spectrum analyzers via GPIB or by translation of disk based data. STAR*Modal*® uses the FRF measurements to identify the modal parameters of the test structure. The mode shapes can be displayed in animation.

---

[1]The STAR System™ User Manual, and The STAR System™ Reference Manual, published by GenRad.

There are many steps in a modal test. This appendix outlines the main ones.

CALIBRATION

Prior to any test, the transducers have to be calibrated. A relative calibration of each accelerometer and force transducer pair is generally acceptable, and is often superior to absolute calibration of each transducer separately.

SET UP A TEST

This includes defining the project in the computer, laying out the test points (coordinates) on the structure, attaching transducers and setting up the analyzer.

STRUCTURAL GEOMETRY

The geometry and coordinate locations are defined in the program. The display sequence used for the animation is also entered.

MAKE MEASUREMENTS

The FRFs between all coordinates and the reference coordinate (either fixed excitation point, or fixed response point, depending on the test procedure) are measured. Depending on the analyzer in use, the STAR*Modal*® program may offer limited control, to facilitate data capture. For this project the STAR*Modal*® program partially controlled the Hewlett Packard 3562A analyzer.

ANALYSIS

Identify modal peaks, and set cursor bands. For each band, identify and set the number of modes, determine the most suitable curve fit method, fit all measurements. This is where the "art" becomes as important as the "science".

VIEW RESULTS

Display animated mode shapes, inspect tables of natural frequencies and viscous damping ratios. Verify repeatability and "reasonableness" of the results.

DOCUMENT RESULTS

STAR*Modal*® offers a variety of standard plots of measurements and mode shapes. However, since it is a Windows compatible program, the results are better presented by transferring them to other processing programs, such as spreadsheets and word processors.

PLEASE CHECK THE APPROPRIATE BLOCK BELOW:

-AQ # ____________

☐ ____________ copies are being forwarded. Indicate whether Statement A, B, C, D, E, F, or X applies.

☑ DISTRIBUTION STATEMENT A:
APPROVED FOR PUBLIC RELEASE: DISTRIBUTION IS UNLIMITED

☐ DISTRIBUTION STATEMENT B:
DISTRIBUTION AUTHORIZED TO U.S. GOVERNMENT AGENCIES ONLY; (Indicate Reason and Date). OTHER REQUESTS FOR THIS DOCUMENT SHALL BE REFERRED TO (Indicate Controlling DoD Office).

☐ DISTRIBUTION STATEMENT C:
DISTRIBUTION AUTHORIZED TO U.S. GOVERNMENT AGENCIES AND THEIR CONTRACTORS; (Indicate Reason and Date). OTHER REQUESTS FOR THIS DOCUMENT SHALL BE REFERRED TO (Indicate Controlling DoD Office).

☐ DISTRIBUTION STATEMENT D:
DISTRIBUTION AUTHORIZED TO DoD AND U.S. DoD CONTRACTORS ONLY; (Indicate Reason and Date). OTHER REQUESTS SHALL BE REFERRED TO (Indicate Controlling DoD Office).

☐ DISTRIBUTION STATEMENT E:
DISTRIBUTION AUTHORIZED TO DoD COMPONENTS ONLY; (Indicate Reason and Date). OTHER REQUESTS SHALL BE REFERRED TO (Indicate Controlling DoD Office).

☐ DISTRIBUTION STATEMENT F:
FURTHER DISSEMINATION ONLY AS DIRECTED BY (Indicate Controlling DoD Office and Date) or HIGHER DoD AUTHORITY.

☐ DISTRIBUTION STATEMENT X:
DISTRIBUTION AUTHORIZED TO U.S. GOVERNMENT AGENCIES AND PRIVATE INDIVIDUALS OR ENTERPRISES ELIGIBLE TO OBTAIN EXPORT-CONTROLLED TECHNICAL DATA IN ACCORDANCE WITH DoD DIRECTIVE 5230.25, WITHHOLDING OF UNCLASSIFIED TECHNICAL DATA FROM PUBLIC DISCLOSURE, 6 Nov 1984 (Indicate date of determination). CONTROLLING DoD OFFICE IS (Indicate Controlling DoD Office).

☐ This document was previously forwarded to DTIC on ____________ (date) and the AD number is ____________.

☐ In accordance with provisions of DoD instructions, the document requested is not supplied because:

☐ It will be published at a later date. (Enter approximate date, if known).

☐ Other. (Give Reason)

DoD Directive 5230.24, "Distribution Statements on Technical Documents," 18 Mar 87, contains seven distribution statements, as described briefly above. Technical Documents must be assigned distribution statements.

Lawrence E. Clemens
Print or Type Name

410-293-6926
Telephone Number

[signature]
Authorized Signature/Date

TOTAL P.02